Input choice under carbon constraint

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Abstract

This paper assesses the impact of emission trading on short-term input demand as well as on long-term production decisions, tacking into account the possibility of input substitions. After setting a simple model based on standard production theory and aimed at carachterizing the virtual threshold price such that the long term capacity choice is optimal, we discuss the impact of input price uncertainty. Firms decisions will depend on the complex interplay between three effects. First, the "average effect", due to the carbon price, causes a decrease in the input capacity with respect to a reference case where the permits market does not exist. Second, the "marginal effect" or the impact of price variability, which instead leads to an expansion of the installed equipment. Third, in the short term, these decisions interact with the "technology effect", i.e. substitution between the polluting and clean inputs. Models simulations show that this interaction can result in weak emission reductions.

1 Introduction

Most of the literature concerning the effect of environmental policy on firms' long term decisions focuses on cleaner technology innovation and diffusion and compares different policy approaches, generally arguing that market-based instruments are superior to command-and-control regulation [1]. Less is known on the interaction between end-of-pipe abatement decisions and long term investment in productive capacity. In fact, in the short-term, only marginal changes in the production technology are feasible. When a market for permits like the European Trading Scheme (henceforth, EU ETS) is introduced, each regulated firm faces a basic choice between buying (or selling) allowances, and using alternative technologies to reduce emissions. Pollution can be offset in two ways. First, by lowering the output scale. Second, the production process or the inputs used, such as fuels for instance, can be altered. Large firms, in order to accomplish the already existing severe European environmental regulations, have mostly reached high environmental standards either in production processes or in reducing the offending gas released as a by-product in the air. Therefore, due to such advanced technological situation, coupled with a characteristic inelastic demand for particular products such as electricity or ore-mining materials,[2] abating by reducing the output scale can be considered as the exception rather than the rule.[3]

In the EU ETS, the largest affected sector that received the lower amount of initial permits, i.e. the fuel-burn energy producers, can use a cheapest short-term (or end-of-pipe) abatement alternative, i.e. fuel-switching. This process involves the replacement of high–carbon (sulphur) fuels with low–carbon (sulphur) alternatives. There is empirical evidence that during the EU ETS Phase I (i.e. from 2005 to 2007), carbon price has induced some emissions abatement, in the form of intra-fuel substitution (brown to hard coal) in Germany and improved CO2 efficiency in the UK (Convery et al, 2008). Delarue et al. (2008) calculate the allowance cost necessary to switch a certain coal-fired plant with a certain gas-fired plant in the merit order, hence the allowance cost obtained is a so called switching point. When comparing historic European Union Allowance (EUA) prices in 2005 with the

[1] Some papers consider only the incentives for research and innovation while others study the diffusion of the new technology (Downing and White, 1986; Fisher et al. 1998, Jung et al., 1996; Milliman et al. 1989). Other papers focus on the timing of environmental policy and show that, in some cases, this timing influences the ranking of the alternative instruments (Jaffe et al., 2002 provide an extensive survey on technological change and environmental problems). Another strand of the literature is concerned with the impact of technological change on marginal abatement cost, in a macroeconomic perspective (for a survey, see Baker et al, 2007).

[2] The largest allocations correspond respectively to iron and steel producers, non-metallic mineral producers and energy producers

[3] See Szabo et al. (2006) for a more comprehensive discussion.

corresponding historic switching points after a simulation work, they found that the EUA prices were high enough to cause a certain switch in the summer season. This finding leds to the use of switching points in establishing allowance cost profiles for several scenarios. Similarly, Denny *et al.* (2009) argue that the CO2 emissions trading system require power generation companies to internalize their emissions costs, because it altered the marginal cost of generators such that heavy polluters became more expensive than lighter polluters. Therefore, they hold that the carbon reduction mechanism could cause coal-fired units to become relatively more expensive compared to gas-fired units. Thus coal units might be shifted from operating continuously as base-load plants to operating on the margin. Cycling costs and carbon savings are compared on the electricity system in Ireland, with the finding that CO2 reduction benefits are outweighed by the added cycling costs. These costs are significantly affected by underlying assumptions regarding generator behavior, demand levels, fuel prices, etc. show that abatement depends not only on the price of allowances, but also on the load level of the electric system and the ratio between natural gas and coal prices. Considine *et al.* (2009) examine the demand for carbon permits, carbon based fuels, and carbon-free energy for 12 European countries after the introduction of the EU ETS. A short-run restricted cost function is estimated in which carbon permits, high-carbon fuels, and low-carbon fuels are variable inputs, conditional on quasi-fixed carbon-free energy production from nuclear, hydro, and renewable energy capacity. The results indicate that prices for permits and fuels affect the composition of inputs in a statistically significant way. The estimates suggest that for every 10 percent rise in carbon and fuel prices, the marginal cost of electric power generation increases by 8 percent in the short run.

Several empirical papers on carbon price drivers have tested whether the switching price that makes an electricity producer indifferent in using coal or gas[4] can be considered a significant regressor,(for an extensive survey on

[4]Formally the switching price (*switch* in €/ton CO_2) is a result of the following relationship: $e_g \times switch + eff_g \times gas = e_c \times switch + eff_c \times coal$, where e_g,e_c measure the emission intensity of gas and coal respectively and eff_g, eff_c the efficiency rate of gas-fired and coal plants. In the above relationship, the LHS measures the marginal cost of producing electricity with a gas-fired plant in a carbon-constrained framework and the RHS is the same for a coal-fired unit. Profit maximizing power producers use the least marginal cost technology. Assuming that each operator has the possibility to switch from high to low cost inputs, fuel price differentials affect the technology used to produce electricity,

this topic, see Bonacina et al. 2009). Despite some mixed results on the significativity of the switching variable, fuel price models form an intrinsic part of carbon price description (Fehr et al., 2006).

We believe that carbon abatement by input-switching is an intriguing topic. However, the very few models dealing with this kind of short-term abatement have at least three main drawbacks.

First, in a carbon constrained economy the right to pollute is a new production factor which comes into play. Traditional approaches disregard the effect of emission allowances on inputs' demand. A partial exception is Jouvet et al. (2005, 2007), who endogenize the technological dimension and analyze inter-industry redistribution of production and inputs (capital and labour). Nevertheless the authors, by opting for a macroeconomic approach, let implicit and perfectly mobile input demand to adjust accordingly. A more micro-economic approach can be found in Newbery (2008), who demonstrates that in the EU ETS, fixing the quantity rather than the price of carbon reduces the price elasticity of demand for gas, amplifying both the market power of gas suppliers, and the impact of gas price increases on the electricity price. This contribution shows important short term effects, but neglects how in turn a modified demand for gas can affect firms investment or equipment decisions in using such less polluting fuel.

Second, the literature on abatement assumes perfect information. Indeed uncertainty has been introduced in other domains related to environmental policies.[5] However, there has been no attempt to consider the effect of both fuel and permit price uncertainty on firms' production decisions in terms of input demand.

Third, the implicit rational for fuel switching is the perfect substitutability of inputs. If this hypothesis can be acceptable if referred to electrical utilities that switch from cola to gas, it seems to us that this simplification is unsatisfactory, notably if we enlarge the analysis to the combination of energy and non energy inputs in the production function. Technological constraints are likely to influence the extent of substitution possibilities.

The key point of our analysis is to assess the impact of emission trading on short-term input demand as well as on long-term production decisions,

and consequently fuel demand.

[5] Uncertainty ranges from agency problems with asymmetric information due to time and/or regulatory ambiguity (Laffont and Tirole, 1996; Farzin and Kort, 2000), to welfare maximization issues with uncertain expected benefits/damages (Baker et al., 2006; Baker, 2007), encompassing endogenous or exogenously driven technological uncertainty.

tacking into account the possibility of input substitutions. However, issues of how to induce long-term technological change are beyond the scope of this paper, which addresses only one aspect of the firm-level response to a CO_2 price.

Our model extends De Villemeur et al. (2006). This latter, by merging traditional investment theories with studies on optimal behavior under uncertainty, consider a two-stage production decision model. In the first period, under the assumption of input prices uncertainty, agents make their optimal investment decision and put production capacity in place. The selected equipment determines the maximum amount of input each firm will be able to use in the second stage where, after uncertainty is resolved, input demand is set. Therefore rationing results from the capacity choice of imperfectly informed operators. Furthermore, the separation of (long-term) investment decisions, which are set consistently with expected state of natures, from short-term inputs' demand, which are defined in certain but constrained environments, provides micro-foundations for short-term cost inefficiencies. This line of research shares similar motivations with the literature on investment (Abel et al.,1996, Dixit and Pindyck 1994) and on firm behavior under uncertainty (Sandmo 1971, Wolak and Kolstad 1991).

We study the effect of enriching the portfolio of available inputs to a representative firm, following the line of research suggested Jouvet et al. (2005, 2007). In addition to a couple of commodities, which we intend without loss of generality, as carbon-intensive energy and a clean (carbon-free) energy, we assume that operators must hold a proper amount of emission allowances as well. In the first stage, competitive and risk neutral firms invest in capacity and in the second how to use their inputs, according to a specific technology. The capacity decision takes place and crucially depends on the input price uncertainty, as the price of the polluting input is random. Environmental regulation is achieved through a market for pollution permits. Therefore, the unit cost of the carbon intensive fuel includes the permits price multiplied by its emission intensity. The permits price shifts the fuel price distribution up.

Uncertainty on the input price plays a key role, even though we do not consider the problem of the investment optimal timing (thus the option value of waiting). We find an oversizing effect with respect to the capacity level that would have been set absent uncertainty. This is a consequence of the irreversibility in choosing the input installed capacity. However, carbon markets, increasing on average the threshold price such that the capacity constraint holds, reduce the oversizing effect and the optimal installed capacity shrinks with respect to a "business as usual" scenario without carbon price. Moreover, we show that firms react to uncertainty by leaving some capacity unused but available if the uncertainty becomes high. Optimal capacity is thus increasing with respect to the input price volatility.

Firms' capacity decisions will depend on the complex interplay between three effects. First, the "average effect", due to the carbon price, causes a decrease in the input capacity with respect to a reference case where the permits market does not exist. Second, the "marginal effect" or the impact of price variability, which instead leads to an expansion of the installed equipment. This effect is amplified when the carbon price is itself random. Third, in the short term, these decisions of course interact with the "technology effect" and the extent of substitution between the polluting and the clean input.

To illustrate these trade-offs, we simulate our model under both the case of a deterministic and random CO_2 price, assuming normal distributions. As preliminary results, we show that the virtual price is increasing with respect to the average of the distribution (the average effect) and decreasing with respect to its standard deviation (the marginal effect). We find that the sensitivity to input price standard deviation increases with the elasticity of substitution (the technology effect). The level of capacity is constant for the case of strict complementarity; at the opposite, in the case of perfect substitution, the optimal capacity is either 0 or strictly positive, given that input demand is necessarily a corner solution. As for pollution, for the set of parameters used in the simulation, we calculate that the emissions reduction amounts at around 10% with respect to a case where there is no carbon price, under the assumption of a Cobb Douglas production function. When the technology effect allows stronger substitutability, more intense emission reduction obtains. Moreover the reduction of emissions is larger when the price of CO_2 is known compared to the case where it is a random variable. As uncertainty in CO_2 price increases the input price variability, the marginal effect dominates and optimal capacities becomes larger. This in turn leads to an increase of input demand and thus emissions.

The paper is organized as follows. Section 2 introduces the two stages of the producer's behavior, that is short run input demand and long run capacity choice under risk neutrality. The role of uncertainty is discussed in Section 3. Several results on the production technology and substitutability are discussed in Section 4. Model simulations (Section 5) illustrate solutions for optimal capacities, input usage and CO2 emissions. We briefly conclude by evoking some policy implications and direction for further research.

2 The model for producer's behavior

In the first stage, a representative risk neutral firm invests in capacity and in the second it decides how to use inputs, according to a specific technology. In the first stage, the capacity decision takes place and crucially depends on the input price uncertainty. In what follows, we analyze the cost minimization problem, at given capacity, and then we endogenize the capacity choice.

2.1 The short run input demand function

We consider the simple case of a production technology with two inputs where input 1 requires some equipment whose size, denoted by $\overline{x_1}$, represents a capacity constraint ($x_1 \leq \overline{x_1}$). Note that input use and input capacity are expressed in the same unit [6].

We consider the case where two inputs are essential; moreover, to simplify the model, we assume that the input x_2 is unconstrained. The production function $y = f(x_1, x_2)$ behaves according to standard regularity conditions.

We assume first that the cost of the equipment depends only on its size and second, that the marginal cost by unit of capacity of the equipment is constant[7] and equal to c_1. Input 1 is a polluting input. Total emissions are equal to ex_1, where e is the per unit emission of 1. The allowed quotas Q is assumed to be such that $Q < e\overline{x_1}$. The price of a permit is denoted by Π. The total unit cost of input 1 is therefore $P_1 + e\Pi$, which we denote $p_1$from now on. Input 2 is a clean input and its price P_2 is normalized to 1.

[6] For example, electric power is produced by combining fuel and equipement, and the range of available equipement can be fully defined by the equipement's size (KW capacity) and the equipement's fuel requirements. Ignoring labor requirements, maintenance requirements and the fact that heat rate varies nonlinearly with the percent of plant capacity used at any instant, the ex post production function for a plant can be defined by a fixed coefficient prodution function. See Stewart (1979) for a more general discussion of this point.

[7] This is the final simplification. Firms could choose simultaneously the quantity and the quality of capital good. See Muller (2000) and their extension of the putty-clay model of capital and energy of Atkeson and Kehoe (1999).

In the *ex post* program corresponding to the second period, the firm minimize short-run costs given input capacity constraint and the realized price for input 1. At this stage, the model describes the firm behavior under an exogenous input quantity constraint, as in Squires (1994). The *ex post* cost minimization program is

$$\underset{x_1,x_2}{Min} \quad (P_1 + e\Pi)x_1 + x_2 + c_1\overline{x_1} =$$
$$p_1x_1 + x_2 + c_1\overline{x_1}$$
$$\text{Subject to } y = f(x_1, x_2),\ x_1 \leq \overline{x_1} \text{ and } Q < e\overline{x_1}.$$

Following the virtual prices approach by Neary and Roberts (1980), the optimal input demand will be

$$\begin{cases} \begin{array}{ll} \overline{x_1} \\ \underline{x_2} & \text{if } p_1 \leq \eta_1, \\ x_1^*(p_1, y) \\ x_2^*(p_1, y) & \text{if } p_1 > \eta_1. \end{array} \end{cases} \tag{1}$$

The variable η_1 is the virtual price of input 1 at which the unconstrained demand for input 1 is exactly equal to $\overline{x_1}$. A complete characterization of the virtual price can be found in Lee and Pitt (1987) in particular it could be written as a function $\eta_1(\overline{x_1}, y)$. Finally $\underline{x_2}$ follows directly from the production function constraint $y = f(\overline{x_1}, \underline{x_2})$[8].

The following figure illustrates the solution. In the short term the maximum amount of input 1 the firm can use is limited by $\overline{x_1}$ and the isoquant is an arc with an extrema points $\left(\overline{x_1}, \underline{x_2}\right)$. For a given capacity level, the ability to switch between the two inputs depends on the realized price and is limited. We distinguish substitution possibilities and switching capacity simply to keep in mind that the marginal rate of technical substitution is a local measure while the switching capacity is a global one and represents the extent of substitution possibilities in the short run.

[8] A formal presentation is $\underline{x_2} = x_2(\overline{x_1})$ such that $y \equiv f(\overline{x_1}, \underline{x_2})$. Note that we have $\frac{d\underline{x_2}}{d\overline{x_1}} = -\frac{\partial f(\overline{x_1}, \underline{x_2})/\partial \overline{x_1}}{\partial(\overline{x_1}, \underline{x_2})/\partial \underline{x_2}} = -\eta_1$. This follows directly from the definition of the virtual price which is defined as an implicit function $\eta_1(\overline{x_1}, y)$ by the restriction that it is the price which would induce an unconstrained firm to purchase the ration level $\overline{x_1}$. As a consequence at this point, the marginal technical rate of substitution is equal to $-\eta_1$.

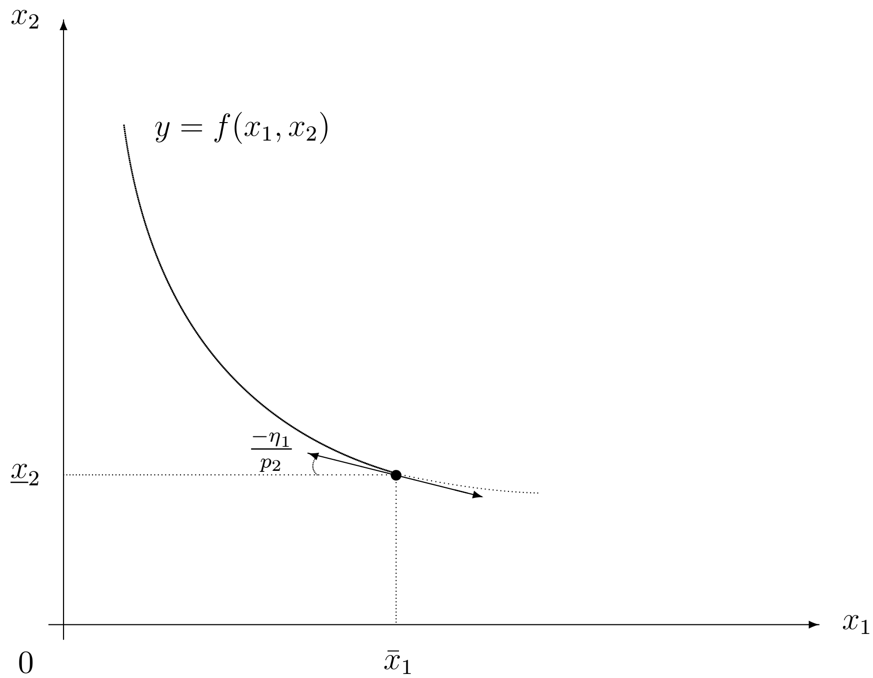


Figure 1

2.2 The long run optimal capacity program under risk neutrality

Recall that input 1 only necessitates some equipment with fixed size; the price $p_1 = P_1 + e\Pi$ is random. Notice that for the moment we assume that the permits price is known and deterministic. Considering p_1 as a random variable with density function $\phi(p_1)$, cumulative density function $\Phi(p_1)$ and with $p_1 \in [0, +\infty[$, the first period program including environmental cost is

$$\begin{aligned} &\min_{\overline{x_1}} \quad \int_0^{\eta_1} \left[(P_1\overline{x_1} + \underline{x_2}) + \Pi(e\overline{x_1} - Q)\right] \phi(p_1)dp_1 + \\ &\int_{\eta_1}^{+\infty} \left[(P_1 x_1^* + x_2^*) + \Pi(e x_1^* - Q)\right] \phi(p_1)dp_1 \\ &+c_1\overline{x_1}. \end{aligned}$$

Let us remark that depending on the value of input price p_1 and hence the optimal level of input use x_1^*, the firm will sell or buy permits depending on the effective emission level. If $ex_1^* > Q$ the firm needs to buy $\Pi(ex_1^* - Q)$. On the contrary, if $ex_1^* > Q$ the firm can sell $\Pi(Q - ex_1^*) = -\Pi(ex_1^* - Q)$. In fact in all the case the situation is such that the firm perceive a fixed revenue ΠQ and pay for effective emission Πex_1^*. The only aspect that changes with respect of a cost minimization program without CO_2 prices is that the price of input 1 is $P_1 + \Pi e$ instead of simply p_1. Notice that with respect to a carbon tax, the equilibrium level of costs (and profits as well, being the output fixed) will be different as the tradable allowances could either generate a deficit or a surplus. However, both a carbon tax and a permit price will shift the input price from P_1 to a larger price by a constant term.

For each possible capacity level the firm faces two cases depending on the *ex post* realized price. If the relative price of input 1 is low enough the firm would use a huge amount of this input but the firm may be constrained by the installed input capacity level. Otherwise, if the price is high there is in some sense a *reserve capacity,* the optimal level of input demand being lower than the input capacity constraint.

The firm's trade-off is simple, by choosing a large capacity $\overline{x_1}$ the firm is able to reach allocative efficiency for a large range of possible prices p_1 but this capacity has a cost.

The first order condition is

$$\int_0^{\eta_1} (p_1 + \frac{\partial x_2}{\partial \overline{x_1}})\phi(p_1)dp_1 + c_1 = 0.$$

Simple manipulation of the first order condition leads to the following equation providing the price threshold η_1 such that the capacity choice is optimal:

$$\Phi(\eta_1)\left[\eta_1 - \mathbb{E}(p_1 \ / p_1 \leq \eta_1)\right] = c_1. \tag{2}$$

3 The role of uncertainty

In this section, we will show how price uncertainty affects the choice of the input capacity level.

First, let $\overline{x_1}^c$denote the optimal input capacity level under certainty. Suppose that the price is known and equal to μ. The cost minimization program of the firm could be simplified in this case in one step only

$$\underset{x_1,x_2,\overline{x_1}}{Min} \quad \mu x_1 + x_2 + c_1\overline{x_1},$$

$$\text{Subject to } y = f(x_1, x_2),\ x_1 \leq \overline{x_1}.$$

Clearly the input capacity constraint will be binding because there is no gain to hold some reserve capacity in this case. As a result the optimal solution for the demand and capacity for input 1 are the same and equal to

$$\overline{x_1}^c = x_1^*(\mu + c_1, y).$$

We now prove the following proposition.

Proposition 1 *The input capacity chosen by a risk neutral firm* $\overline{x_1}^*$*, will exceed* $\overline{x_1}^c$*.*

Proof. Suppose that while the price $p_1 = P_1 + \Pi e$ is random with $\mathbb{E}(p_1) = \mu$, the firm decides to fix the input capacity at $\overline{x_1}^c$.

Then the marginal gain to increase marginally the capacity $\overline{x_1}^c$ is

$$(\mu + c_1)\int_0^{\mu+c_1} \phi(p_1)dp_1 - \int_0^{\mu+c_1} p_1\phi(p_1)dp_1$$

which is the same expression as before, evaluated at the particular case where the virtual price $\mu + c_1$ is associated to the assumed input capacity constraint $\overline{x_1}^c$.

This marginal gain is equal to

$$\begin{aligned} & (\mu + c_1)\left[1 - \int_{\mu+c_1}^{+\infty} \phi(p_1)dp_1\right] - \mu + \int_{\mu+c_1}^{+\infty} p_1\phi(p_1)dp_1 \\ = \; & c_1 + \int_{\mu+c_1}^{+\infty} (p_{1-}(\mu + c_1))\phi(p_1)dp_1. \end{aligned}$$

Provided that the distribution probability is not degenerated over $[\mu+c_1, +\infty[$ which means that with a non zero probability the realized price could be greater that $\mu + c_1$, then the integral in the marginal gain is positive and so the marginal gain greater than the marginal cost of the capacity c_1. Using the fact, as shown earlier, that the marginal gain is decreasing with respect

to the capacity it follows that the firm increases the capacity level above $\overline{x_1}^c$.
■

The intuition behind Proposition 1 is that when faced with a random price of input, the risk-neutral firm sees the possibility of a lower production cost if the realized price is low and if the input capacity do not limit the use of this input. With the option to use input 1 up to the input capacity level, the firm seeks low production costs during low input price by choosing an input capacity greater than $\overline{x_1}^c$. Looking at the results of a model with incremental investment (Pindyck, 1988), we see here a different investment strategy. Instead of reducing the amount of installed capacity with respect to the optimal input capacity level under certainty, uncertainty leads to to 'oversize' a project, which is a consequence of the once and for all decision. This effect is similar to the one obtained by Hartl and Kort (1996) and Dangl (1999), where uncertainty in future demand leads to an increase in optimal installed capacity. Firms react to uncertainty by leaving some capacity unused but available if the uncertainty is very high.

Proposition 1 has two interesting consequences. The first one is that since the optimal input capacity with price uncertainty is greater than $\overline{x_1}^c$, it follows that the virtual price solution of the first order condition associated to the ex ante cost minimization program is necessarily lower than $\mu + c_1$. The second one is that the expected cost with price uncertainty will be below the cost calculated under the price $p_1 = \mu$.

By denoting by $\mathbb{E}(C)$ the ex ante expected cost, we can calculate the impact on the ex ante expected cost of the threshold price η :

$$\left.\frac{\partial \mathbb{E}(C)}{\partial \eta_1}\right|_{\eta_1=\mu+c_1} = \frac{\partial \eta_1}{\partial \overline{x_1}} \left[\int_0^{\mu+c_1} (p_1 - (\mu + c_1))\phi(p_1) dp_1 + c_1 \right].$$

Using the same simplifications as those in proposition 1 concerning the integral in the LHS, it is easy to show $\left.\frac{\partial \mathbb{E}(C)}{\partial \eta_1}\right|_{\eta_1=\mu+c_1} \geq 0$. It follows that since the virtual price decrease from $\mu + c_1$, so does the expected cost. Then the optimal cost with price uncertainty is smaller than the optimal cost without uncertainty and the same expected input prices.

Now, we will study the effect of a marginal increase in uncertainty.

We consider a modified cost minimization program of the firm where the price of input 1 is now equal to $\gamma p_1 + \theta$.

It is easy to show that *ex post* optimal demand for inputs are

$$\begin{cases} \begin{array}{ll} x_1^*(\gamma p_1+\theta,y) & \text{if } p_1 > \frac{\eta_1-\theta}{\gamma}, \\ x_2^*(\gamma p_1+\theta,y) & \\ \overline{x_1} & \text{if } p_1 \leq \frac{\eta_1-\theta}{\gamma}, \\ \underline{x_2} & \end{array} \end{cases}$$

The first order condition associated to the ex ante cost minimization with respect to the input capacity could now be written as

$$\Phi(\frac{\eta_1-\theta}{\gamma})\left[\eta_1-\mathbb{E}(\gamma p_1+\theta\ /p_1\leq\frac{\eta_1-\theta}{\gamma})\right]=c_1.$$

Proposition 2 *The input capacity is a decreasing function of the expected input price.*

Proof. Taking the last condition as an identity at the optimal input capacity, we can show that if the expected price of input 1 increases then the firm will choose a lower $\overline{x_1}^*$.

$$\frac{\partial\eta_1}{\partial\overline{x_1}}\frac{\partial\overline{x_1}}{\partial\theta}d\theta=\int_0^{\frac{\eta_1-\theta}{\gamma}}\phi(p_1)dp_1.$$

Provided that $\eta_1-\theta$ is positive the RHS is positive. Using the fact that $\frac{\partial\eta_1}{\partial\overline{x_1}}\leq 0$ the result follows immediately. ■

We will consider now the effect of an increase in the variability of the density function of the price in terms of a mean preserving spread.

$$d\mathbb{E}(\gamma p_1+\theta)=\mu d\gamma+d\theta=0\Leftrightarrow\frac{d\theta}{d\gamma}=-\mu.$$

Differentiating the first order condition totally with respect to γ and using the condition about the scale parameter of the price distribution leading to an unchanged mean price, we obtain

$$\frac{\partial\eta_1}{\partial\overline{x_1}}\frac{\partial\overline{x_1}}{\partial\gamma}d\gamma=\frac{\int_0^{\frac{\eta_1-\theta}{\gamma}}(p_1-\mu)\phi(p_1)dp_1}{\int_0^{\frac{\eta_1-\theta}{\gamma}}\phi(p_1)dp_1}$$

From proposition 1 we know that the optimal input capacity is larger than the certain case evaluated at the expected price which is $\gamma\mu+\theta+c_1$. A consequence of this result is that the virtual price associated to the optimal capacity level is lower than this value.

Corollary 3 *Absent carbon markets, in the business as usual scenario the input capacity chosen by a risk neutral firm is above* $\overline{x_1}^*$.

Proof. Immediate, as the carbon market shifts the distribution of the price P_1 by a constant Πe; therefore, $E(P_1) < E(p_1)$. ∎

Carbon markets, increasing on average the price η_1 thus reduce the oversizing effect and the optimal installed capacity shrinks.

As for the impact of price variability, the following holds.

Proposition 4 *The input capacity is an increasing function of the variability of price.*

Proof. We need to proof the negative sign of the integral in the numerator of the RHS in previous equation. Remark first that this integral is an increasing function with respect to η_1. Second, from proposition 1 we know that the optimal input capacity is larger than the certain case evaluated at the expected price which is $\gamma\mu + \theta + c_1$. A consequence of this result is that the virtual price associated to the optimal capacity level is lower than this value.

Evaluating this integral at $\eta_1 = \gamma\mu + \theta + c_1$,leads to

$$\int_0^{\mu+\frac{c_1}{\gamma}} (p_1 - \mu)\phi(p_1)dp_1.$$

This integral could be written as

$$-\int_{\mu+\frac{c_1}{\gamma}}^{+\infty} (p_1 - \mu)\phi(p_1)dp_1,$$

which is clearly negative. ∎

As long as the carbon price is deterministic, this second effect only depends from the uncertainty on the input price P_1. An interesting extension of the model is to consider a stochastic permits price that could amplify the incentive to increase capacity as a strategy to cope with a stronger variability of the input price. Therefore, it is interesting to compare the "average effect", which causes a decrease in the input capacity, and the "marginal effect" that instead leads to an expansion of $\overline{x_1}$. We discuss these cases in the model simulation under a Normal Law (see Section 5)

4 The technology and the role of input substitution

We solve our model in the particular case of a CES production function. Some of the following results could be derived from a more general production function. Nevertheless, as we will focus mainly on the role of substitution possibility between inputs, and show its great importance in the choice of equipment capacity as well as for CO_2 regulation, the model is solved for this particular functional form, which is

$$y = \gamma \left[\delta x_1^{\rho} + (1-\delta) x_2^{\rho}\right]^{1/\rho},$$

where y represents output, and x_i for $i = 1, 2$ represents input usage.

The CES production function is defined for $\rho \in]-\infty, 1]$, and $0 \leq \delta \leq 1$. Moreover we know that the CES production function leads to the Leontieff production function, as $\rho \to -\infty$, the Cobb-Douglas production function as $\rho = 0$, and the linear production function, as $\rho = 1$. We denote by $\sigma = \frac{1}{1-\rho}$ the substitution elasticity between the two inputs.

For the CES technology, the virtual price associated to the input capacity constraint is

$$\eta_1 = \left[\left(\frac{y}{\gamma \overline{x_1}}\right)^{\rho} - \delta\right]^{\frac{1-\rho}{\rho}} (1-\delta)^{\frac{-1}{\rho}} \delta.$$

Denoting by η_1^* the solution of equation 2, the technology plays now a role in the optimal capacity level which is determined by

$$\overline{x_1}^* = \frac{y}{\gamma} \left[\delta + (1-\delta)\left(\frac{(1-\delta)\eta_1^*}{\delta}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{-1}{\rho}}. \tag{3}$$

Equation 3 shows that the optimal capacity could be expressed as the demand for input 1 evaluated at a particular relative price η_1^* which does not depend on the elasticity of substitution. As a consequence, comparing optimal capacities levels which follow from different CES functions distinguished by the values of the elasticity of substitution σ only, is rather simple. Optimal capacity satisfies the standard equality between the MRTS at the point $\left(\overline{x_1}, \underline{x_2}\right)$ and the relative price η_1^*,

$$MRTS = \frac{d\underline{x_2}(\overline{x_1})}{d\overline{x_1}} = -\eta_1^*.$$

Input ratio. The input ratio $\frac{\overline{x_1}}{\underline{x_2}}$ can be expressed as a function of σ.In fact, basic algebra for the CES production function with 2 factors, gives

$$\frac{d\underline{x_2}(\overline{x_1})}{d\overline{x_1}} = -\frac{\delta}{1-\delta}\left(\frac{\underline{x_2}}{\overline{x_1}}\right)^{1/\sigma},$$

and thus we have

$$\frac{\overline{x_1}}{\underline{x_2}} = \left(\frac{1-\delta}{\delta}\eta_1^*\right)^{-\sigma}.$$

The sign of the derivative of the input capacity ratio with respect to the substitution elasticity, could be easily calculated.

$$\frac{\partial\frac{\overline{x_1}}{\underline{x_2}}}{\partial\sigma} = -\ln\left(\frac{(1-\delta)\eta_1^*}{\delta}\right)\left[\frac{(1-\delta)\eta_1^*}{\delta}\right]^{-\sigma}$$

It follows that the input ratio $\frac{\overline{x_1}}{\underline{x_2}}$ is increasing (decreasing) with respect to σ when η_1^* is larger than (respectively lower) than $\frac{\delta}{1-\delta}$.

Optimal capacity. The optimal capacity $\overline{x_1}$ can also be expressed as a function of σ.The relationship between the optimal level of the energy equipment capacity for input 1 and the parameters of the technology, is not straightforward in this case.

Deriving $\overline{x_1}^*$ with respect to the substitution elasticity σ leads to the following non-linear expression,

$$\frac{\partial\overline{x_1}^*}{\partial\sigma} = \frac{\overline{x_1}^*}{1-\sigma}\left[\frac{1}{\sigma}\ln(\frac{\gamma\overline{x_1}^*}{y}) - \sigma\ln(\frac{(1-\delta)\eta_1^*}{\delta})(1-\delta(\frac{\gamma\overline{x_1}^*}{y})^{\frac{\sigma-1}{\sigma}}\right].$$

It is easy to show that if $\frac{(1-\delta)\eta_1^*}{\delta} = 1$, then $\overline{x_1}^* = \frac{y}{\gamma}$ which is independent of σ. Moreover we know that optimal input capacity is $\overline{x_1}^* = \frac{y}{\gamma\delta}$ when the production function is linear ($\sigma \to \infty$) and $\eta_1^* < \frac{\delta}{1-\delta}$.

Similarly we have $\overline{x_1}^* = \frac{y}{\gamma}$ when the production function is Leontieff ($\sigma = 0$).

To understand the shape of the optimal input capacity as a function of σ, it is convenient to find, depending on σ, the value of the MRTS leading to a fixed capacity $\overline{x_1}^* = \frac{y}{\gamma\delta}$ (solution when goods are perfects substitutes and $\eta_1^* < \frac{\delta}{1-\delta}$). We must find $MRTS(\sigma)$ such that

$$\frac{y}{\gamma}\left[\delta + (1-\delta)\left(\frac{1-\delta}{\delta}MRTS(\sigma)\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} = \frac{y}{\gamma\delta}.$$

Solving previous equation gives,

$$MRTS(\sigma) = \frac{\delta}{1-\delta}\left[\frac{\delta\left(-1+\delta^{-\frac{1}{\sigma}}\right)}{1-\delta}\right]^{\frac{1}{\sigma-1}}.$$

Since we have

$\lim_{\sigma\to 0} MRTS(\sigma) = 0,$

$\lim_{\sigma\to +\infty} MRTS(\sigma) = \frac{\delta}{1-\delta},$

and, $MRTS'(\sigma) > 0$

then $\exists\ \widetilde{\sigma} \in]0;+\infty)$ such that $MRTS(\widetilde{\sigma}) = \frac{\eta_1^*}{P_2}$, with $P_2 = 1$.

Given η_1^*,and considering the case where $\eta_1^* < \frac{\delta}{1-\delta}$, then the capacity $\overline{x_1} = \frac{y}{\gamma\delta}$ is optimal for a particular value of the substitution elasticity $\widetilde{\sigma}$ corresponding to a CES production function in between the linear technology and the Leontief production function.

As a consequence the optimal input capacity first increase and then decrease as the substitution elasticity increases. Notice that $\underline{x_2}$ is always decreasing.

Figure 2 illustrates this result where the heavy curve represents the optimal capacity $\overline{x_1}$ for different values of the substitution elasticity. In this figure we consider the case $\eta_1^* < \frac{\delta}{1-\delta}$, the other one could be obtained by symmetry.

Let us remark that for the Cobb-Douglas case, the optimal input capacity is not necessarily larger than $\frac{y}{\gamma\delta}$. For $\sigma = 1$,

$$\overline{x_1}^*_{\sigma=1} = \left(\frac{1-\delta}{\delta}\eta_1^*\right)^{\delta-1}.$$

The optimal capacity is such that as $\eta_1^* \to 0$ then $\overline{x_1}^*_{\sigma=1} \to +\infty$ and as $\eta_1^* \to \frac{\delta}{1-\delta}$ then $\overline{x_1}^*_{\sigma=1} \to \frac{y}{\gamma}$.

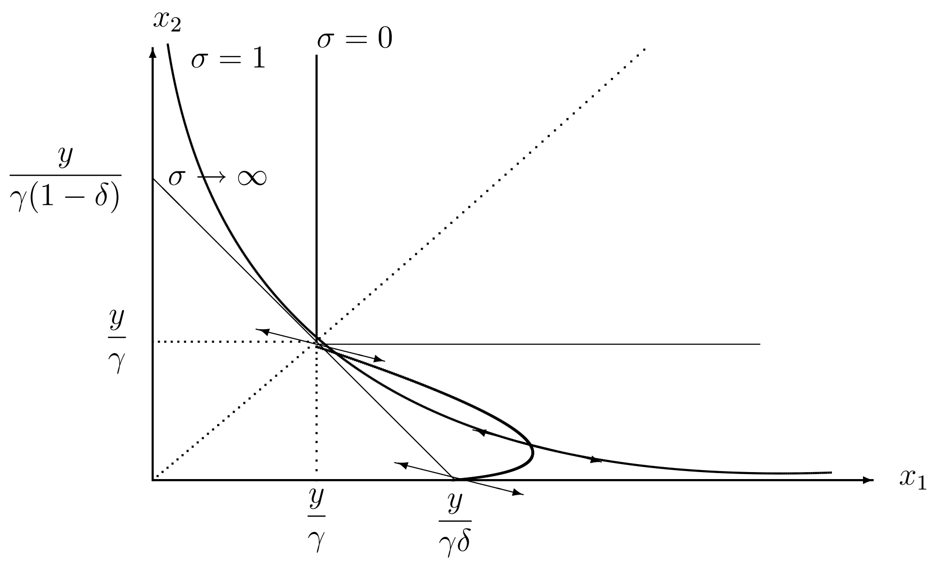


Figure 2. Optimal capacity and $MRTS$.

5 Model simulation

5.1 Optimal virtual price

The following figure 3 represents the solution for

$$\mathbb{P}(p_1 \leq \eta_1)\left[\eta_1 - \mathbb{E}(p_1 \ /p_1 \leq \eta_1)\right] = c_{1,}.$$

when p_1 follows a normal distribution $N(m, s^2)$.

Denoting respectively by ϕ the probability density function for normal distribution and by Φ its cumulative distribution function, we have

$$\mathbb{E}(p_1 \ /p_1 \leq \eta_1) = m - s\frac{\phi(\frac{\eta_1 - m}{s})}{\Phi(\frac{\eta_1 - m}{s})},$$

for the expected price associated to the truncated normal distribution from above with threshold η_1.

The equation to be solved is after simplification

$$(\eta_1 - m)\Phi(\frac{\eta_1 - m}{s}) + s\phi(\frac{\eta_1 - m}{s}) = c_1.$$

The set of parameters are $c_1 = 0.1$, $m \in [1; 1.5]$ and $s \in [0; 0.5]$.

Figure 3 illustrates the different solutions obtained numerically. Notice that η_1^* is increasing with respect to m and decreasing with respect to s, coherently with the results of Propositions 2 and 3.

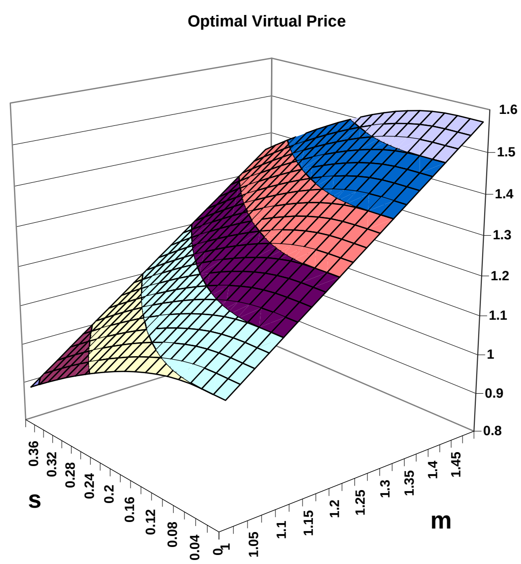


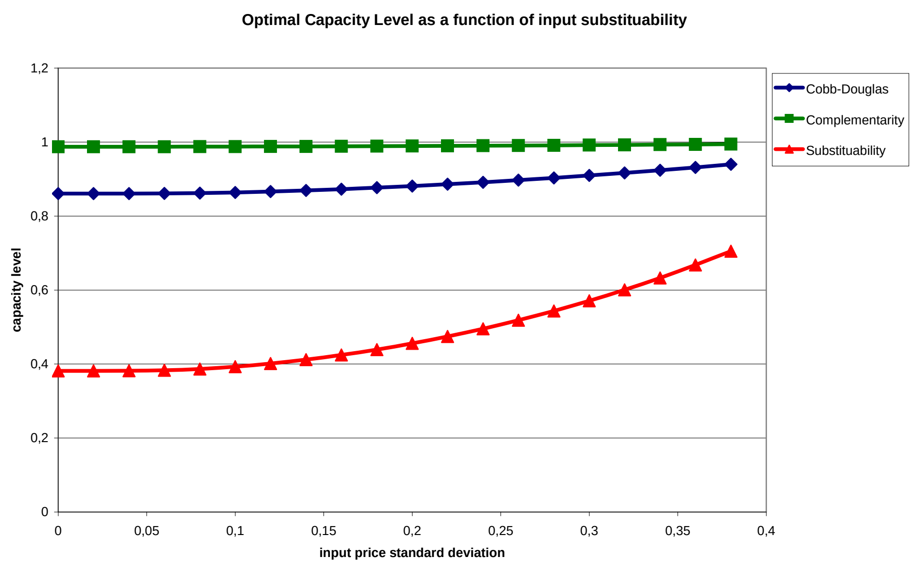
Figure 3.

5.2 Optimal Capacity

The following graph represents the optimal capacity obtained from different assumptions about the input substitution elasticity. The Cobb-Douglas case corresponds to $\sigma = 1$ in between the two other cases we consider are $\sigma = 0.1$ called complementarity and $\sigma = 5$ called substitutability.

The input price follows a normal distribution. In the following figures the expected price for input is fixed at $m = 1.25$.

Optimal capacity levels are increasing with respect to the input price standard deviation. As expected, the sensitivity to input price standard deviation increases with the elasticity of substitution. The level of capacity is constant for the case of strict complementarity as inputs must be used in fixed proportion. On the contrary in the case of perfect substitution the optimal capacity is either 0 or strictly positive, given that input demand is necessarily a corner solution.

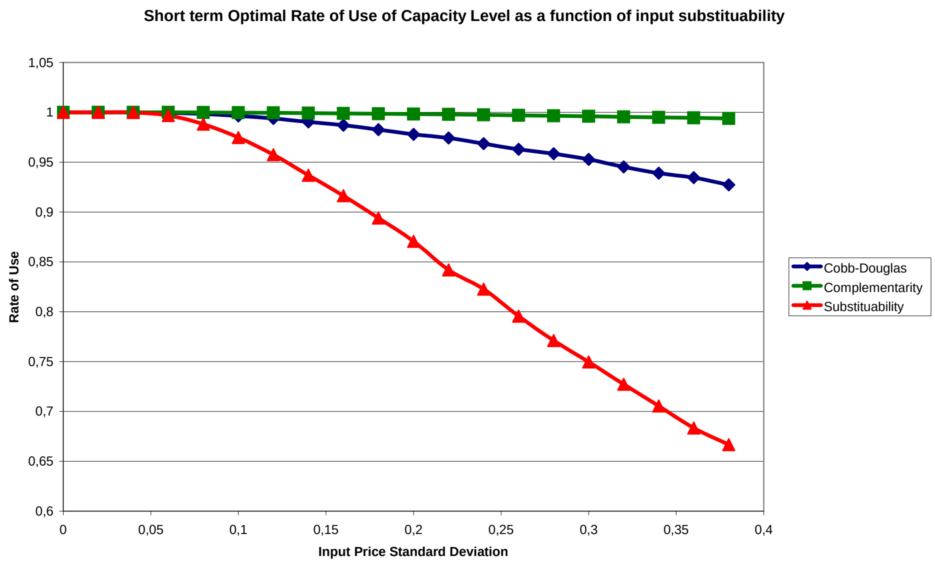


Of course in the short term the emission level will depend on the optimal level of capacity. Let us remark that in this model the expected short term rate of use of the capacity is less than 1 at the optimum. Figure 4 represents the expected rate of use of the capacity for the three case considered.

The usage rate of capacity is computed as:

$$\frac{\mathbb{E}(x_1^*)}{\overline{x_1}^*} = \frac{\mathbb{P}(p_1 \leq \eta_1)\overline{x_1}^* + \mathbb{P}(p_1 > \eta_1)\mathbb{E}(x_1^* \,/ p_1 > \eta_1)}{\overline{x_1}^*},$$

which corresponds to the ratio of expected optimal input demand and the optimal capacity level. As input demands are not constrained by capacity in the short run, the rate of use of capacity is less than 1 at the optimum, except for very low input price standard deviations.



Notice that the optimal rate of use of capacity decreases with respect to input price standard deviation. This is a direct consequence of the marginal effect of uncertainty that decreases the virtual price η_1 and thus increases the optimal capacity.

5.3 CO2 emissions

As last step of the model simulation, we take into account the level of emissions. We consider the case of a fixed and known price for CO_2 $\Pi = 0.4$, and the case where the price for CO_2 is also random, with $\Pi = 0.4 + 0.2\varepsilon$, where ε follows a $N(0, 1)$. Following the same steps as before we determine successively the optimal price limit, the optimal capacities and expected demand to determine the level of emissions. We normalize to 1 the emission level obtained when emissions are free (which corresponds to $\Pi = 0$), under the assumption that the emission intensity is equal to 1.

We represent hereafter relative emissions for a fixed and random CO2 price and compare them to the free emission case.

Of course in the complementarity case, as capacities and input demand are imposed by the technology, there is almost no possibility to reduce emissions.

For the set of parameters used for this simulation, we obtain that the reductions of emissions amounts at around 10% with respect to the emissions-free scenario, if the technology is Cobb-Douglas, while this reduction could be more important for greater elasticity of substitution between the two inputs.

Moreover the reduction of emissions is larger when the price of CO_2 is known compared to the random case. As uncertainty in CO_2 price increases the input price variability, optimal capacities becomes larger which leads to an increase of input demand and thus emissions.

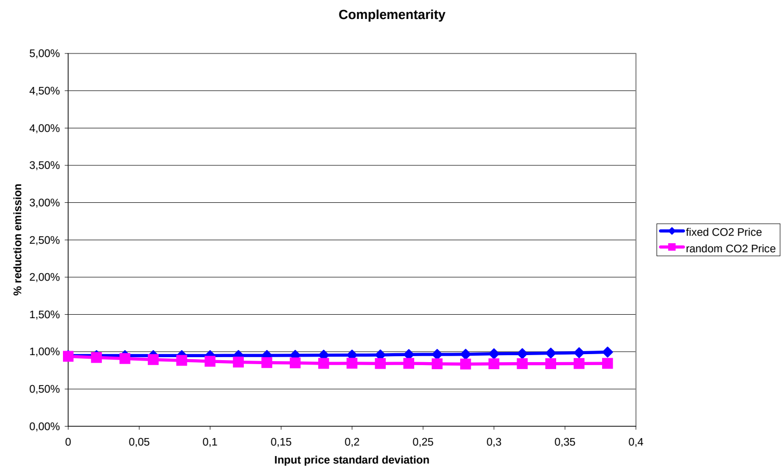


Figure 6a

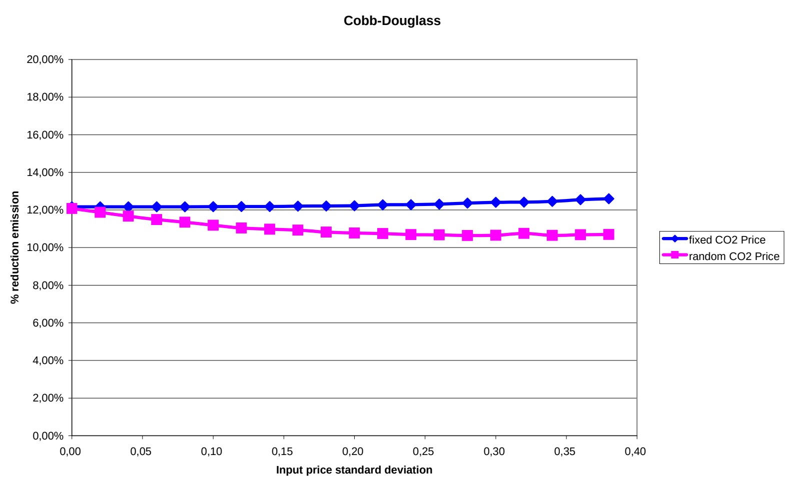


Figure 6b

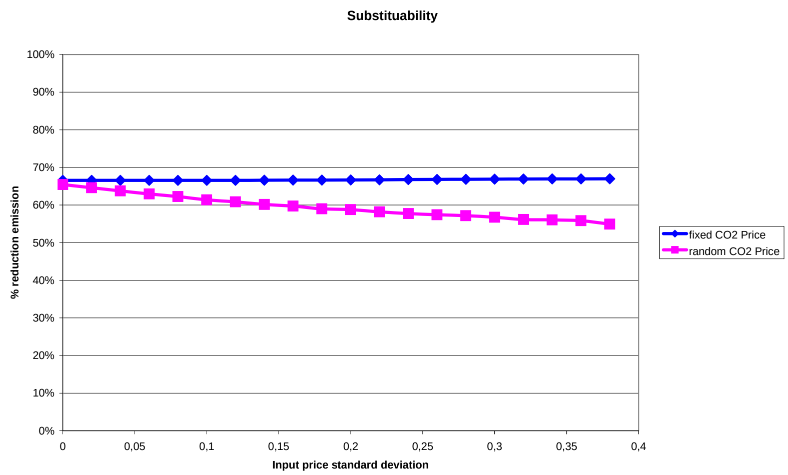


Figure 1: Figure 6c

6 Conclusions

Our model shows that the interaction between production decisions and carbon constraints is very complex, as long as both short-term and long-term strategies are involved. Taking into account input substitutability, our approach clarifies the impact of the fuel-switching cost on permit price, which in theoretical and empirical models is almost undetectable. From an environmental policy perspective, our results show that imposing specific targets for emission reductions without intervening on capacity choices (as for instance subsidies or R&D incentives to clean technologies) and relying on carbon markets only can lead to inefficiencies. Moreover, if the permit price is too volatile, as it can be the case when uncertainty in environmental policy is high, these inefficiencies amplify and emission reductions are weaker. To better understand this latter effect, further research will be devoted to endogenize the equilibrium permits price, by introducing some degree of firms heterogeneity.

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