Bilevel Optimization for Natural Gas and Other Infrastructure Management

Keynote Presentation at Workshop on
The Economics of Natural Gas, New Research Areas for a Reconfigured Gas Scene
Paris-Dauphine University, Place du Maréchal de Lattre de Tassigny 75016 PARIS

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Outline

1. Overview
2. The Role of Bilevel Optimization for Improved Natural Gas and Other Infrastructure Management
    1. The Role of Bilevel Optimization & Natural Gas
    2. The Role of Bilevel Optimization & Other Infrastructure Management
3. Optimization & Equilibrium Problems, Formulations and Examples
4. Detailed Energy Infrastructure Investment Example: Top-Level Player is an Energy Company
5. Summary & References

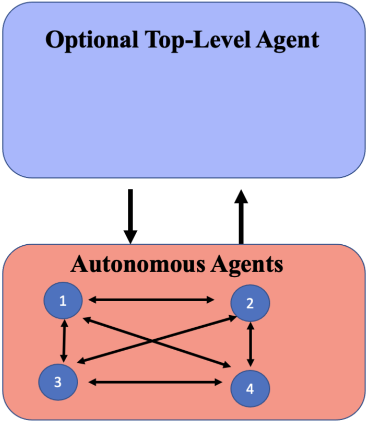
Optimization and Data Analytics for Improving Social Welfare

- Use technology, management and economics to improve social welfare and achieve societal sustainability goals
- One key area to concentrate on is infrastructure, e.g., energy, transportation, water, the environment
- By improving these key areas, society as a whole, as well as individuals will see an improved quality of life
- If we consider just energy as an example (but also true of other areas of infrastructure), no "silver bullet" here, society will need a portfolio of options for
    - supply/demand (e.g., renewable but intermittent supply, demand response)
    - transmission/distribution
    - regulatory/policy incentives

  to achieve the above goals
- Will need a combination of new technologies as well as market equilibrium-based and optimization-based models to maximize benefits given limited resources and competing goals (i.e., multiobjectve, user vs. system equilibria)

"30,000-foot" / "10,000-meter" Perspective: Modeling and Analysis of Data-Driven Systems with Autonomous Agents

Top-level agent sets policy or incentives/coordinates activities of autonomous agents



Top-level agent could also be a private company with an advantageous position

Each agent solves a specific optimization problem, potentially interacting with other agents

The resulting "system" behavior is not known *a priori*

Bilevel Optimization Problem (or Mathematical Program with Equilibrium Constraints) [Gabriel et al., 2012]

$$\min f(x,y)$$
$$\text{s.t. } (x,y)\in\Omega$$
$$y \in S(x)$$
where

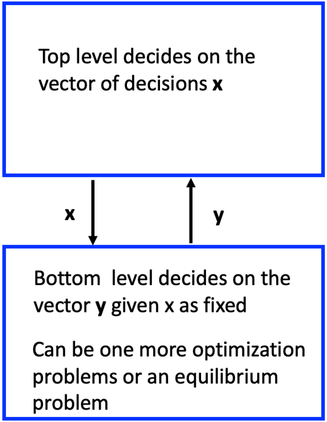
Ω set of constraints for (x,y)

$x \in R^{n_x}$ upper-level variables

$y \in R^{n_y}$ lower-level variables

$f(x,y)$ upper-level objective function

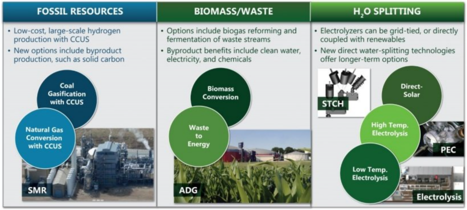
$S(x)$ solution set of lower-level problem (opt. or game)



Outline

Example #1: Hydrogen Focus

- "Hydrogen Shot": U.S. Dept. of Energy, produce hydrogen at 2 USD/kg by 2025 and 1 USD/kg by 2030 via net-zero-carbon pathways
- Currently most hydrogen in the U.S. is produced by large-scale natural gas reforming without CCS– good to reach cost targets but prefer low-carbon pathways
- Production of hydrogen from natural gas costs about 1.50 USD/kg, clean hydrogen about 5 USD/kg
- In the near- and mid-term, electrolysis (electricity to split water into hydrogen and oxygen) anticipated to start reaching cost targets
- In the mid-to long-term, use waste streams and others based on solar energy are expected to become viable
- Government (as top-level player) can have a role with research funding, tax reductions, carbon credits, promote CCS, etc. to incentivize these hydrogen-production goals

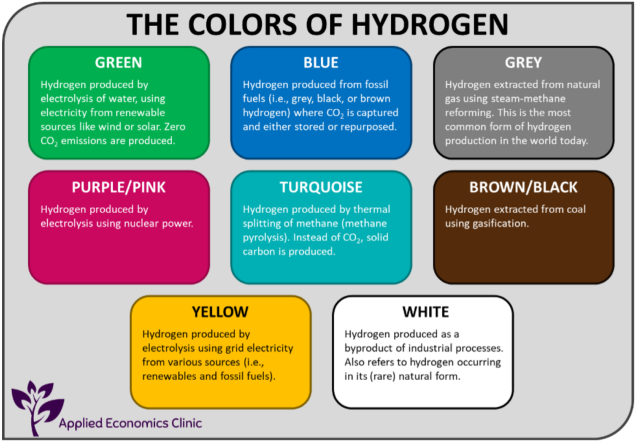


CCUS: carbon capture, utilization, and storage; SMR: steam methane reforming; ADG: anaerobic digester gas; STCH: solar thermochemical hydrogen; PEC: photoelectrochemical

https://afdc.energy.gov/fuels/hydrogen_basics.html, https://www.energy.gov/eere/fuelcells/hydrogen-production-pathways, https://www.cnbc.com/2022/01/06/what-is-green-hydrogen-vs-blue-hydrogen-and-why-it-matters.html

The "Colors" of Hydrogen

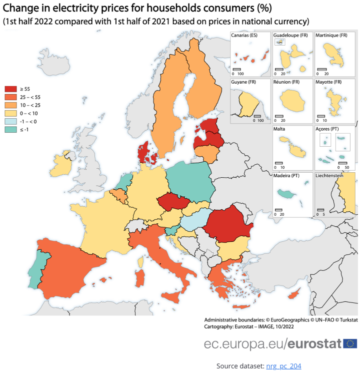
- Many different "colors" of hydrogen

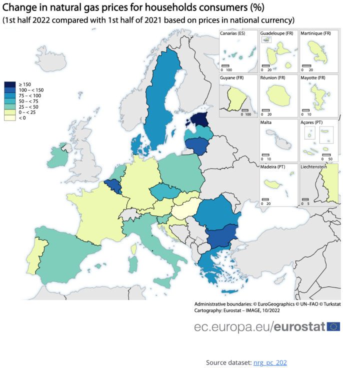


https://aeclinic.org/aec-blog/2021/6/24/the-colors-of-hydrogen

Example #2 :Liquefied Natural Gas (LNG) & European Energy Security Focus

- Due to Russian war in Ukraine and other market-related influences, surge in power and gas prices in Europe





https://ec.europa.eu/eurostat/web/products-eurostat-news/-/ddn-20221031-1

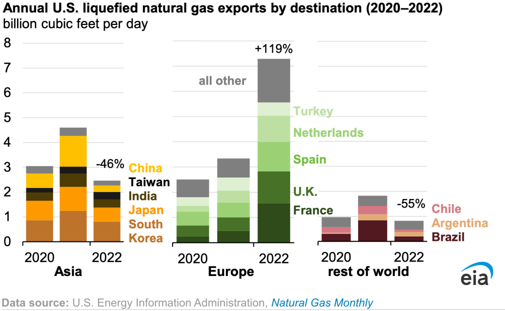
Energy Security & Natural Gas

- Examples of 3 Possible Ways to Improve Security Relative to Natural Gas:
  1. Increase liquefied natural gas (LNG) imports, requires building more terminals and/or new contracts, improving regulatory process
  2. Diversify pipeline gas suppliers (e.g., away from Russia), requires building new pipelines and/or new contracts
  3. Being more efficient about natural gas usage (e.g., reducing demand)
- Will concentrate on item #1

https://ec.europa.eu/info/news/focus-reducing-eus-dependence-imported-fossil-fuels-2022-apr-20_en

U.S. LNG Exports to Europe

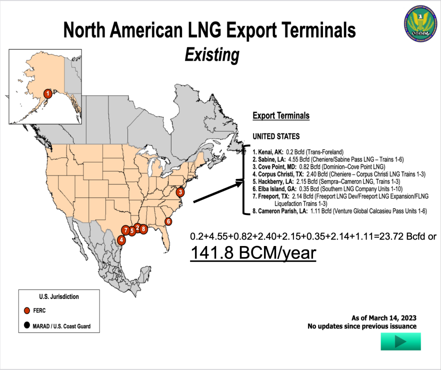
- In 2022, U.S. LNG exports to Europe increased by 141%(increase of 4.0 Bcf/day) compared with 2021
- U.S. LNG exports to Europe in 2022 was 64% of total exports
- Four main European countries receiving U.S. LNG: France, United Kingdom, Spain, Netherlands (combined, 74%) of U.S. LNG exports to Europe
- In 2022, Europe Increased LNG imports to the highest-ever 14.9 Bcf/day, 65% higher than in 2021
- In 2022, huge reductions of U.S. LNG to Asia (46% decrease overall, 78% decrease for China)



U.S. Energy Information Administration, Today in Energy

U.S. LNG Export Capacity to Europe

- Total U.S. LNG Export Capacity (mostly Gulf of Mexico, as of 14 March 2023)
  - Existing terminals: 141.8 Bcm/year
  - Approved, not yet built : 302.1 Bcm/year
  - Proposed: 100.8 Bcm/year



https://www.csis.org/analysis/how-us-lng-could-help-europe-and-climate#: :text=The, https://www.reuters.com/business/energy/could-us-ship-more-lng-europe-2022-03-25/, https://www.ferc.gov/natural-gas/lng

U.S. LNG Exports to Europe

- Joint U.S.-E.U. understanding on natural gas and renewable energy
- The European Commission to work with EU member states to accelerate regulatory procedures to review for LNG import infrastructure
- Joint "Task Force on Energy Security," (President Joe Biden, European Commission President Ursula von der Leyen showed on March 25).
- According to the White House, the joint task force will work "to ensure energy security for Ukraine and the EU in preparation for next winter and the following one, while supporting the EU's goal to end its dependence on Russian fossil fuels."

https://www.powermag.com/u-s-agrees-to-ramp-up-lng-exports-to-europe-actively-reduce-natural-gas-demand/#: :text=For

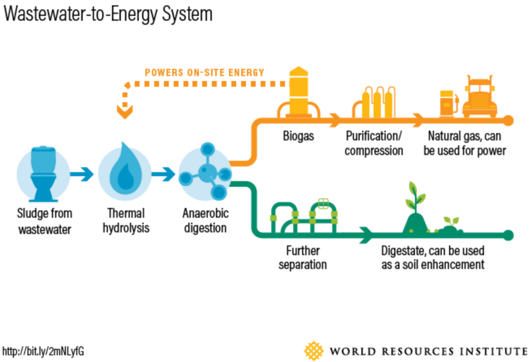
U.S. LNG Exports to Europe

- Charlie Riedl, executive director of the Center for Liquefied Natural Gas (CLNG), a trade group comprising all aspects of the U.S. LNG supply chain.
  - "The LNG industry can build, but regulators must do their part to help expedite the essential infrastructure that is needed here and in Europe to meet these ambitious goals and help our European allies," Riedl
  - the EU (as top-level player) could 'accelerate the regulatory approval process' and support long-term contracting mechanisms with U.S. LNG suppliers. That will 'send a strong signal to our allies in Europe that they can count on U.S. LNG to help with energy security and climate leadership well into the future,' Riedl
- Nikos Tsafos (James R. Schlesinger Chair in Energy and Geopolitics with the Energy Security and Climate Change Program at the Center for Strategic and International Studies in Washington, D.C.
  - Now-2030's, U.S. LNG to Europe (and Asia) for filling the gap from Russia
  - 2030's and beyond, U.S. LNG to Asia for lowering carbon emissions
  - Could also use LNG to make hydrogren with carbon capture (blue hydrogen)
- The EU/national governments can foster flexibility in LNG contracting, the government could help here relative to regulatory approval, flexibility in private sector LNG contracting (top-level player major energy company)
- Governments can also specify diversity-of-supply constraints for risk mitigation

https://www.powermag.com/u-s-agrees-to-ramp-up-lng-exports-to-europe-actively-reduce-natural-gas-demand/#: :text=For , https://www.csis.org/analysis/how-us-lng-could-help-europe-and-climate#: :text=The

Example #3 :Wastewater-to-Energy

- Can also get biogas from wastewater, can be purified/compressed for other uses (e.g., power production, compressed natural gas for buses)
- Biogas is thus a "renewable" resource which is non-intermittent and positively correlated with population growth

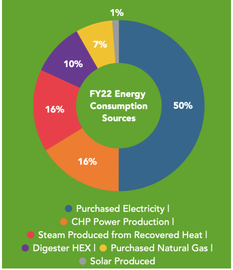


https://www.wri.org/insights/wastewater-best-hidden-energy-source-youve-never-heard#: :text=How%20Does%20Wastewater%20Become%20Energy,being%20released%20into%20the%20atmosphere

Wastewater-to-Energy Advantages

- Self-sufficiency for wastewater treatment plants, not depending on outside power (with possible outages)
- Emissions reductions, using the biogas for energy rather than releasing to the atmosphere
- Waste management instead of dumping/landfilling
- Economic benefits for waste-to-energy operations
- Some selected examples of countries using waste-to-energy: U.S., China, Brazil, Argentina, Norway
- Government as the top-level player, can seek to incentivize greater production of biogas through renewable energy credits (RECs) or other monetary means
- Consider DC Water, the Washington, DC-based water & wastewater utility
- They produce their own power from waste and sell the RECs via biogas generation and heat capture systems (2022: greater than 3.3 million USD in value)

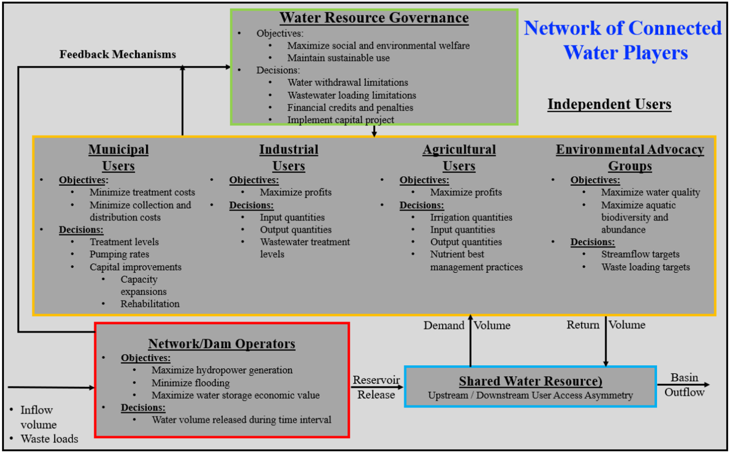
FY22 Actual	FY22 Target	FY23 Target	FY27 Target
% of Renewable Energy			
43%	42%	43%	43%
Energy Produced On-Site (MWh)			
185,500	185,100	185,500	187,000
RECs Sold			
$3.3M	$2.0M	$3.7M	$4.5M
Clean Rivers Tunnel Volume in Service (Million Gallons)			
114 MG	105 MG	105 MG	157 MG



https://www.wri.org/insights/wastewater-best-hidden-energy-source-youve-never-heard#: :text=How%20Does%20Wastewater%20Become%20Energy,being%20released%20into%20the%20atmosphere, https://www.dcwater.com/sites/default/files/customer_care/ESG2023_v1.pdf

Outline

Bilevel Optimization/MPEC Structure Example: River Systems [Boyd et al., 2022]



Bilevel Optimization in Energy: Cutting Across Sustainable Energy Technologies, Markets, and Policy

Top Level

- Design decisions (e.g., what materials, size of CCS plants)
- Dominant firm generation decisions
- Government policy decisions
- Investment decisions for technologies

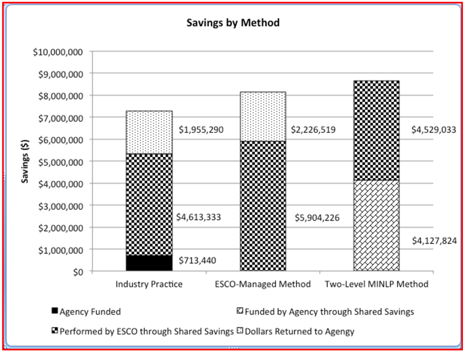
↓ ↑

Bottom Level

- Operational decisions (e.g., how to operate the technologies, the CCS plants)
- Rest of the market (competitive fringe, ISO) generation and endogenous market prices
- Market responses to policy
- Market responses to investments

CCS=Carbon, capture, and sequestration.

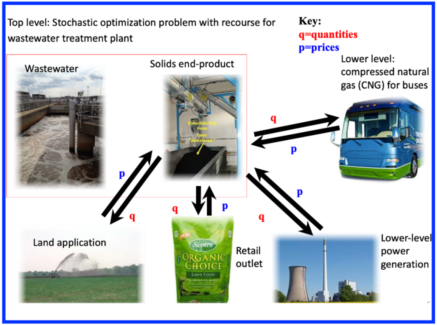
Bilevel Optimization: Energy Conservation Example [B. R. Champion and S.A. Gabriel, 2015]



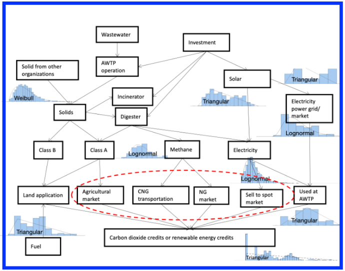
- Energy Conservation Programs
- Two-level optimization model to better manage energy conservation programs for agencies, schools
- More efficient decision-making for internal/outsourced energy project retrofits

Bilevel Optimization: Stochastic Wastewater-to-Energy Example [U-tapao et al., 2016]

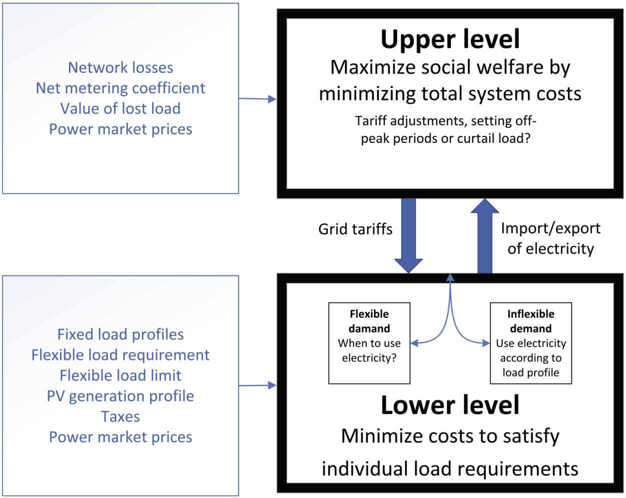
Top and Bottom Levels



Process Diagram for Top Level Wastewater Treatment Plant

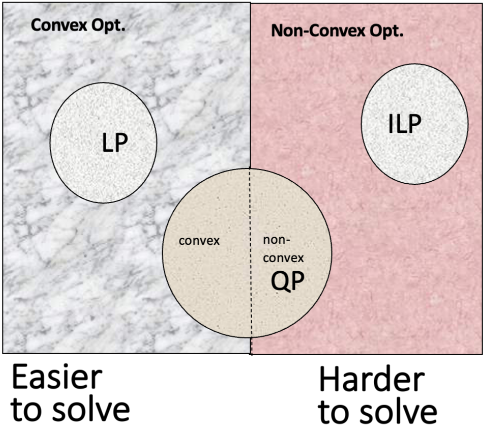


Stochastic Bilevel Optimization: Optimal Grid Tariffs [Askeland et al., 2020]



Overview of Optimization Problems

The Big Picture



KEY

LP=linear program

ILP=integer linear program

QP=quadratic program

1

Complementarity Problems, Mixed Complementarity Problems (MCP) [Gabriel et al., 2012]

(Mixed) Nonlinear Complementarity Problem MNCP

Having a function $F : R^n \to R^n$, find an $x \in R^{n_1},\ y \in R^{n_2}$ such that

$F_i(x,y) \geq 0, x_i \geq 0, F_i(x,y) * x_i = 0 \; for \; i = 1,\ldots,n_1$

$F_i(x,y) = 0, y_i$ free, $for \; i = n_1 + 1,\ldots,n$

Example

$$F(x_1,x_2,y_1) = \begin{pmatrix} F_1(x_1,x_2,y_1) \\ F_2(x_1,x_2,y_1) \\ F_3(x_1,x_2,y_1) \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - y_1 \\ x_1 + x_2 + y_1 - 2 \end{pmatrix}$$ so we want to find x_1, x_2, y_1 s.t.

$$\begin{array}{lll} x_1 + x_2 \geq 0 & x_1 \geq 0 & (x_1 + x_2) * x_1 = 0 \\ x_1 - y_1 \geq 0 & x_2 \geq 0 & (x_1 - y_1) * x_2 = 0 \\ x_1 + x_2 + y_1 - 2 = 0 & y_1 \text{ free} & \end{array}$$

One solution: $(x_1, x_2, y_1) = (0, 2, 0)$, why? Any others?

If all functions (linear) affine, we get the linear complementarity problem (LCP)

Sources for Complementarity Problems: Linear Programming

Consider a (primal) linear program in the variables $x \in R^n$:

$$\begin{aligned} min_x \quad & c^T x && (1a) \\ s.t. \quad & Ax \geq b \qquad (y) && (1b) \\ & x \geq 0 && (1c) \end{aligned}$$

and corresponding dual linear program in the variables $y \in R^m$

$$\begin{aligned} max_y \quad & b^T y && (2a) \\ s.t. \quad & A^T y \leq c \qquad (x) && (2b) \\ & y \geq 0 && (2c) \end{aligned}$$

and complementary slackness for both primal and dual problems:

$$(Ax - b)^T y = 0, (c - A^T y)^T x = 0 \qquad (3)$$

Sources for Complementarity Problems: LP Primal and Dual Feasibility, Complementary Slackness

We can rewrite things a bit to get the following equivalent form. Find $x \in R^n, y \in R^m$ such that:

$$0 \leq c - A^T y \perp x \geq 0 \tag{4a}$$

$$0 \leq Ax - b \perp y \geq 0 \tag{4b}$$

This is exactly the (monotone) linear complementarity problem (LCP) in nonnegative variables (x, y) and is exactly the KKT optimality conditions as applied to the primal LP. Here

$$F(x,y) = \begin{pmatrix} F_x(x,y) \\ F_y(x,y) \end{pmatrix} = \begin{pmatrix} c - A^T y \\ Ax - b \end{pmatrix} \text{ or} \tag{5}$$

$$F(x,y) = \begin{pmatrix} c \\ -b \end{pmatrix} + \begin{pmatrix} 0 & -A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{6}$$

Sources for Complementarity Problems: KKT Optimality Conditions for Nonlinear Programs

Consider a nonlinear program of the following form where $f : R^n \to R, g_i : R^n \to R, i = 1, \ldots, m, h_j : R^n \to R, j = 1, \ldots, p$:

$$
\begin{aligned}
min_x \quad & f(x) && && (7a)\\
s.t. \quad & g_i(x) \leq 0 && i = 1, \ldots, m \quad (\lambda_i) && (7b)\\
& h_j(x) = 0 && j = 1, \ldots, p \quad (\gamma_j) && (7c)
\end{aligned}
$$

The KKT conditions are to find primal variables $x \in R^n$ and Lagrange multipliers $\lambda \in R^m_+$, $\gamma \in R^p$ such that:

$$
\begin{aligned}
0 =& \nabla f(x) + \sum_i \nabla g_i(x)\lambda_i + \sum_j \nabla h_j(x)\gamma_j, x \texttt{ free} && (8a)\\
0 \leq& -g(x) \perp \lambda \geq 0 && (8b)\\
0 =& h(x), \quad \gamma \text{ free} && (8c)
\end{aligned}
$$

Selected Sources for Complementarity Problems: KKT Optimality Conditions for Nonlinear Programs

Putting all these conditions together, we get a mixed complementarity problem of the following form. Find vectors $\lambda \in R^m_+$, $x \in R^n, \gamma \in R^p$ such that:

$$F(x,\lambda,\gamma) = \begin{pmatrix} \nabla f(x) + \sum_i \nabla g_i(x)\lambda_i + \sum_j \nabla h_j(x)\gamma_j \\ -g(x) \\ h(x) \end{pmatrix}$$

with

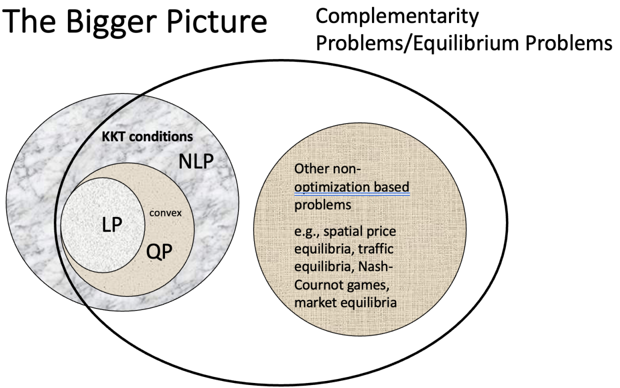
$$0 = F_x(x,\lambda,\gamma) \qquad x \text{ free} \tag{9a}$$

$$0 \leq F_\lambda(x,\lambda,\gamma) \qquad \perp \lambda \geq 0 \tag{9b}$$

$$0 = F_\gamma(x,\lambda,\gamma) \qquad \gamma \text{ free} \tag{9c}$$

$\Rightarrow$ Connection to game theory problems

Overview of Equilibrium Problems: Generalizing Certain Optimization and Game Theory Problems [Gabriel et al., 2013]

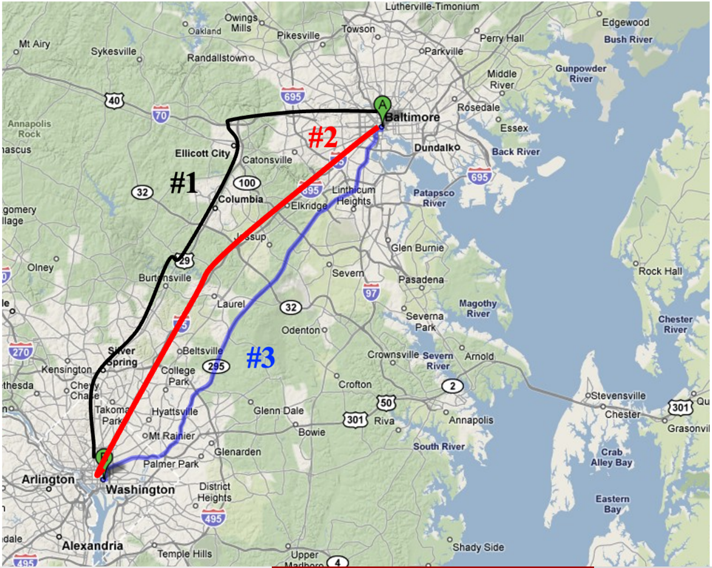


KKT=Karush-Kuhn-Tucker optimality conditions.

Classical Examples of the "Bottom Level" of the Bilevel/MPEC Approach

- Wardrop Traffic Equilibrium Problem
  - originally from [Wardrop, 1952]
  - additive path costs
  - nonadditive path costs
- Spatial Price Equilibrium Problem
  - classical approach using linear programming from [Samuelson, 1952] then [Takayama and Judge, 1964]

Wardrop Traffic Equilibrium [Wardrop, 1952]



A->B,

38.5 miles

62 kilometers

About 53 minutes travel time

Which route to take for commuting?

Black, red, blue?

Wardrop Traffic Equilibrium Principle

- Original formulation [Wardrop Equilibrium,1952] and then [Aashtiani and Magnanti, 1981]
- "At equilibrium, for each origin-destination pair the travel times on all routes serving the same OD pair, actually used are equal, and less than then travel times on all nonused routes."
- Wardrop user equilibrium principle: users will choose the minimum cost path between each OD pair resulting in paths with positive flow all having equal costs, paths with costs higher than the minimum will have no flow.
- Can express this traffic equilibrium problem in some cases in terms of arc flows f as opposed to path flows F.

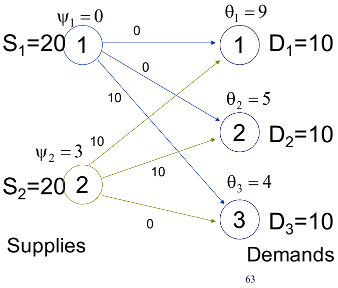
$$
\begin{aligned}
(C_p(F) - u_i)F_p &= 0, \forall p \in P_i, i \in I \\
C_p(F) - u_i &\geq 0, \forall p \in P_i, i \in I \\
\sum_{p \in P_i} F_p - D_i(u) &= 0, \forall i \in I \\
F_p &\geq 0, \forall p \in P \\
u_i &\geq 0, \forall i \in I
\end{aligned}
$$

Nonadditive Traffic Equilibrium Problem [Gabriel & Bernstein, 1997]

- Can extend the basic Wardrop traffic equilibrium problem to include non-additive path costs.
- In this case, the path flows are needed but you can avoid enumerating all the paths via an algorithm.
- The algorithm brings in paths only when an improvement towards finding an equilibrium can be detected by solving a related shortest-path problem.
- Some examples of nonadditive path costs:
  - Nonlinear value of time
  - Nonadditive tolls and fares
  - Pricing emissions fees
  - Congestion pricing

Spatial Price Equilibrium Problem in Power Markets

- Consider the problem of an independent system operator (ISO) that is trying to efficiently run a power network composed of of power generation nodes (S_1, S_2) and demand nodes (D_1, D_2, D_3)
- Cost minimization ("Transportation Problem") with the ISO's decisions x_{ij} the (primal) flow variables, ψ_i, θ_j the dual variables (Lagrange multipliers) at respectively, supply node i and demand node j, not so realistic since ψ_i and θ_j should be elastic not fixed



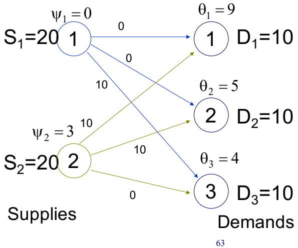
Spatial Price Equilibrium Model: Transportation Problem

$$\min_{x_{ij}} \sum_i \sum_j c_{ij} x_{ij} \tag{10a}$$

$$\text{s.t.} \quad \sum_j x_{ij} \leq supply_i \qquad (\psi_i), \quad \forall i \tag{10b}$$

$$\sum_i x_{ij} \geq demand_j \qquad (\theta_j), \quad \forall j \tag{10c}$$

$$x_{ij} \geq 0 \qquad \forall i, j \tag{10d}$$

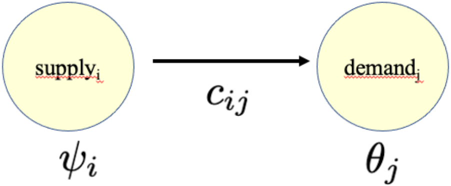


Transportation Problem Formulation, Karush-Kuhn-Tucker Optimality Conditions

$$0 \leq \psi_i + c_{ij} - \theta_j \perp x_{ij} \geq 0, \forall i, j$$

$$0 \leq supply_i - \sum_j x_{ij} \perp \psi_i \geq 0, \forall i$$

$$0 \leq \sum_i x_{ij} - demand_j \perp \theta_j \geq 0, \forall j$$



Spatial Price Equilibrium Problem in Power Markets

- x_{ij}:= flow from supply i to demand j, $S_i := \sum_j x_{ij}, D_j := \sum_i x_{ij}$
- $\psi_i(S_i)$:= inverse supply function
- $\theta_j(D_j)$:= inverse demand function
- c_{ij}:= marginal transport cost

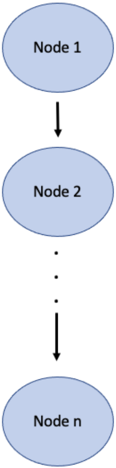
Overall MCP in terms of (nonnegative) flows x_{ij} is thus:

$$0 \leq \psi_i \left(\sum_j x_{ij} \right) + c_{ij} - \theta_j \left(\sum_i x_{ij} \right) \perp x_{ij} \geq 0 \tag{11}$$

- $x_{ij} > 0 \Rightarrow \psi_i \left(\sum_j x_{ij} \right) + c_{ij} = \theta_j \left(\sum_i x_{ij} \right)$ or marginal cost = marginal benefit
- $\psi_i \left(\sum_j x_{ij} \right) + c_{ij} - \theta_j \left(\sum_i x_{ij} \right) > 0 \Rightarrow x_{ij} = 0$ or no flow when marginal cost is higher than marginal benefit

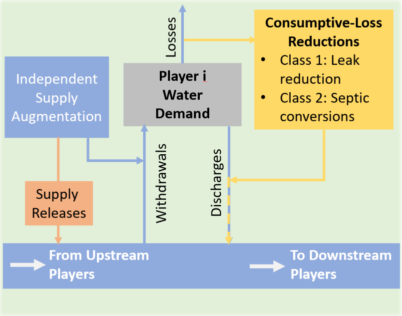
What is Being Modeled: Balancing Water Markets, Spatial and/or Temporal Positional Advantages

- Consider a river network with n users (nodes), $I = \{1, \ldots, n\}$
- Let D_i represent the nodes downstream of node i, U_i represent the nodes upstream of node i
- For all nodes in D_i, would they be willing to pay upstream nodes $i, i-1, \ldots, 1$ something to:
    - Increase volume of water? Demand and supply for water can be **stochastic**.
    - Decrease volume of pollutants? Fate and transport of pollutants can be seemingly **stochastic** in nature.
- If so, how to compute payments?
- Will everyone do better with these water markets?



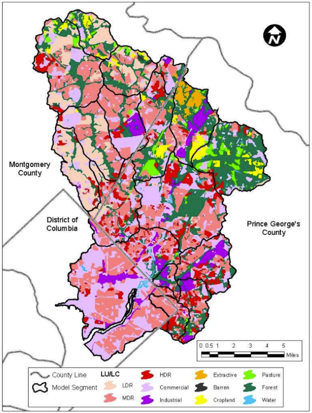
Water Flow in a River-Water Quantity Example [Boyd et al., 2022]

- Generically, each node (player) withdraws/discharges water from the river
- Each player can also add their own supply
- There is an opportunity though to be more efficient by reducing consumptive losses
- Overall, each node solves an optimization problem related to maximizing benefits less costs with possible participation in consumptive loss-reduction markets



Land Use Heterogeneity-Water Quality Example [Boyd et al., 2023]

- Loading heterogeneity in the watershed
- Water runoff and pollutants could vary according to: land use, soil properties, vegegation
- Comparative advantages amount the various players in different regions of the watershed



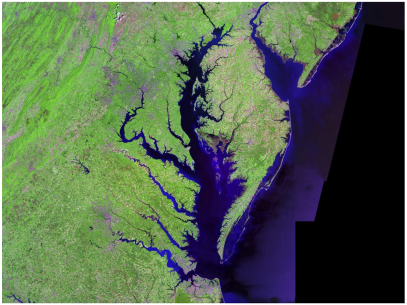
Shultz et al. (2007)

Infrastructure Markets: Incentivizing Water Market Participation

- What we propose for water markets is voluntary participation in the consumptive-loss reduction markets– water quantity (see Boyd et al., 2022)) or pollution-reduction credits (see Boyd et al., 2023))
- This means, "downstream" river users pay "upstream" ones to improves the efficiency of water lost so more water makes it downstream (water quanity) or a similar sort of payment but to reduce pollutants upstream (e.g., sediment deposition, probabilistic processes) for overall river benefit (e.g., better flood control), **TMDL chance constraints (reliability)**
- Each river node will be modeled as solving a particular optimization problem, the concatenation of the resulting Karush-Kuhn-Tucker (KKT) optimality conditions plus system or market-clearing conditions gives rise to a mixed complementarity problem (MCP)
- A solution is flows and prices in the various river nodal markets (and other items)
- This problem then is the "bottom level" of the two-level perspective presented earlier

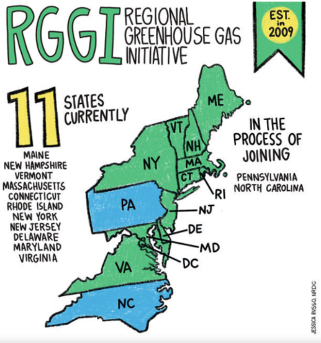
Virginia Nutrient Credit Exchange Association

- There are other markets similar to water with success stories
- Consider the Virginia Nutrient Credit Exchange Association
  - Established in 2005 to reduce nitrogen and phosphorus discharges to the Chesapeake Bay
  - Voluntary collective of owners of 105 wastewater treatment plants (WWTPs)
  - Pollution reduction goals exceeded by over 2,000% for nitrogen and 450% for phosphorus in 2011
  - Smaller WWTPs compensated larger facilities to upgrade on their behalf
  - Pollution levels can be **stochastic** based on many factors



Carbon Allowance Markets: Regional Greenhouse Gas Initiative,

- Regional Greenhouse Gas Initiative (RGGI)
- RGGI is a cooperative, market-based effort among 11 U.S. states with a population of about 60 million to cap and reduce CO2 emissions from the power sector
- As of June 2022, RGGI states raised over $5 billion in carbon allowance auctions for supporting communities for local energy, health and environmental goals
- Carbon prices depending on **stochastic** supply/demand of emitted carbon are **uncertain**

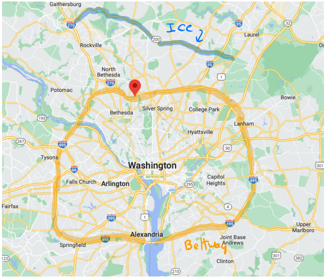


https://www.nrdc.org/resources/regional-greenhouse-gas-initiative-model-nation

https://www.rggi.org

Transportation: Flow-Based Pricing Outside Washington, DC

- Intercounty Connector (ICC), a way to avoid the Washington Beltway, save time
- To use the ICC, you need a transponder in your car, from which flow-based prices are charged
- The payment puts a value on free flow of travel
- There is no obligation to use the ICC, drivers can just use regular (non-tolled) roads and avoid fees
- **Stochastic demand** for the ICC?



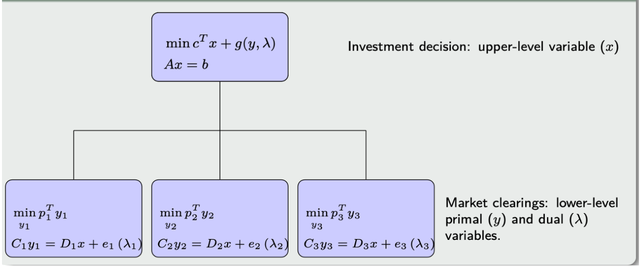
Infrastructure Markets: Incentivizing Market Participation

- Clearly these infrastructure markets can work, the problem is how to incentivize everyone to participate
- Should there be some minimum participation required?
- Should there be legal mandates?
- How to handle the **uncertainty** in the markets/lower-level equilibrium problem?
- Should participation be voluntary based on some social improvement?
- In the a river system context
  - The work by Allen et al. (2022), determines appropriate taxes to "nudge" the user equilibrium (i.e., MCP) towards a social best using inverse optimization and LCP theory
  - The work by Boyd et al. (2023) analyzes water quality issues for the Anacostia River (Washington, DC) using a stochastic LCP formulation involving TMDLs (total maximum daily loadings) for pollutants

Energy Infrastructure Investment Example [Bylling et al., 2019, 2020]

- Application was in power generation/transmission infrastructure investment
- Model and algorithms also applicable to natural gas as well

Bilevel Investment Problem



Mathematical Details-1

Linear Bilevel Programming Problem

$$\min_{x,y,\lambda} c^T x + d^T y^* + k^T \lambda^* + g(y^*, \lambda^*) \quad (1a)$$

$$\text{s.t. } Ax = b \quad (1b)$$

$$x \geq 0 \quad (1c)$$

$$y^* \in \text{argmin}\{p^T y \quad (1d)$$

$$\text{s.t. } Cy = Dx + e \quad (1e)$$

$$y \geq 0\} \quad (1f)$$

$$\lambda^* \in \text{argmax}\{\lambda^T (Dx + e) \quad (1g)$$

$$\text{s.t. } C^T \lambda \leq p\} \quad (1h)$$

$g(y, \lambda)$ is assumed linear in y.

Mathematical Details-2

Linear Bilevel Programming Problem

$$\min_{x,y,\lambda} c^T x + d^T y^* + k^T \lambda^* + \lambda^{*T} M y^* \quad (1a)$$

$$\text{s.t. } Ax = b \quad (1b)$$

$$x \geq 0 \quad (1c)$$

$$y^* \in \text{argmin}\{p^T y \quad (1d)$$

$$\text{s.t. } Cy = Dx + e \quad (1e)$$

$$y \geq 0\} \quad (1f)$$

$$\lambda^* \in \text{argmax}\{\lambda^T (Dx + e) \quad (1g)$$

$$\text{s.t. } C^T \lambda \leq p\} \quad (1h)$$

Replace with $\lambda^T M y$ in most cases

Mathematical Details-3

Reformulate the problem as

$$\min_{x} c^T x + F(x) \tag{2a}$$

$$\text{s.t. } Ax = b \tag{2b}$$

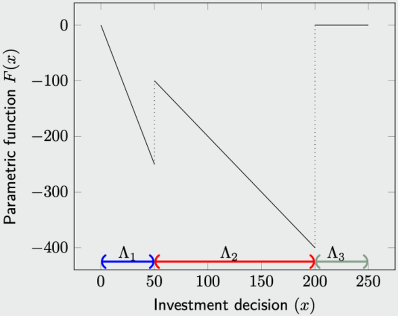
$$x \geq 0 \tag{2c}$$

with

$$F(x) = d^T y(x) + k^T \lambda(x) + \lambda(x)^T M y(x), \tag{3}$$

Mathematical Details-4

Investment Example



Critical regions, Λs, for each lower-level optimal basis.

Proposition

- F is a piece-wise linear function (on *critical regions*).
- F is possibly discontinuous (because of the dual variables).

Mathematical Details-5

Data and approach

- Danish price regions DK1 and DK2 [7].
- Regions connected with 600 MW DC cable.
- Potential generation investment in DK1 and DK2.
- Full year of demand data.

DK1 and DK2

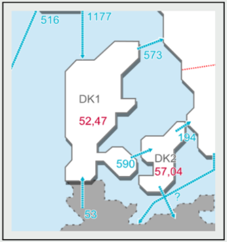
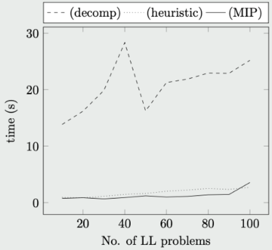


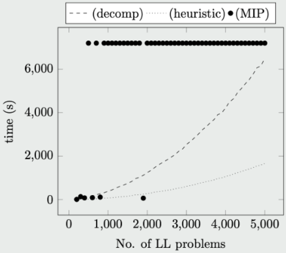
Figure: Source: nordpoolspot.com

Mathematical Details-6

Solution time



(a) No of LL problems from 10 to 100.



(b) No of LL problems from 200 to 5000.

Figure: Solution times for increasing number of LL problems.

Summary

- Natural gas and other infrastructure management can be improved through the use of analysis using bilevel optimization/MPEC models directly taking into account stochastic elements (e.g., at the lower level)
- Users are autonomous agents with a top-level decision-maker testing out various policy regimes to seek overall best policies for social welfare
- Computational issues related to the bilevel structure– these can be overcome through the use of optimization/operations research techniques for small- or medium-scale problems
- For larger problems, there are opportunities for research to improve the related modeling and algorithmic approaches

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